Hedonic Pricing in Realistic Urban Structures, or What if Tiebout Called and Nobody Sorted?

Spencer Banzhaf*
Resources for the Future

November 2003

Early Draft. Conclusions Preliminary.

I. Introduction

The hedonic price regression continues to be a popular method for modeling quality-differentiated goods, with common applications to housing, to recover marginal values for public goods, and to fast-changing products like electronics, to compute quality-adjusted price indices. Because of its intuitive appeal and ease of implementation, the model's popularity continues despite misgivings about its strong assumptions about the continuity of choices available to households in the amenity space.

This paper explores the importance of such assumptions in practice with an application to hedonic housing prices. Using a set of actual housing data to mimic a realistic city, and using a set of known preference orderings, it simulates a housing market to recover equilibrium prices. It then performs hedonic regressions on these prices to assess the quality of the estimated marginal values for amenities, as in Cropper et al. (1988). Despite good fit to the data, the mean marginal values are generally computed with large errors, typically in the range of 25 to 30 percent and sometimes much larger—even though there are no measurement errors or other data problems and no omitted variables. Evidence is presented that, for some amenities with discrete distributions and/or lower priority in preferences, households with different demands cluster on similar amenity levels. This evidence suggests that either not all marginal values can be priced into the equilibrium and/or that they are so discontinuous that even very flexible functional forms cannot identify them. Relaxing the discreteness and structure in the urban environment overcomes these problems, allowing households to sort on

* Resources for the Future, 1616 P Street NW, Washington, DC, 20036. T 202 328-5033, email banzhaf@rff.org. I thank Kerry Smith, Leland Deck, Subhrendu Pattanayak, Randy Walsh, and participants of Camp Resources and the CU workshop of environmental economics for helpful comments and suggestions.

all amenities simultaneously. The results suggest the need for caution in applying the hedonic method for recovering marginal values.

II. The Hedonic Model

The hedonic model treats differentiated products as bundles of their underlying attributes *z* (houses as bundles of rooms, lots, neighborhood characteristics; cars as bundles of horsepower, handling, safety; and so forth).[1] Households bid on the products based on their demand for the amenities offered, sorting themselves by preferred types. This sorting results in an equilibrium price function, $p(z)$, where prices are functions of the underlying attributes and determined by the supply and demand for those attributes. It also implies that the slope of the hedonic price function at a point is equal to the marginal willingness to pay of households located at that point. If households are all identical, it is exactly equal to households' bids (Freeman 1974); if households are heterogeneous, it is an upper envelope to their bids.[2]

The hedonic model has been used both to forecast prices based on characteristics (using the price function) and to estimate implicit prices of amenities (using its derivative). Both approaches have been used to adjust price indices for quality change and new goods and to evaluate changes in public-good amenities for benefit cost analysis. In applications to price indices, the derivative of the price function can be used to adjust prices for the change in quality (the current practice of the US Bureau of Labor Statistics), or the price function can be used to impute what the price would have been if quality had remained constant. In applications to benefit cost analysis, the derivative of the price function recovers marginal values, while movement along the price function can serve as a bound for non-marginal values (Bartik 1988, Kanemoto 1988).

As is well known, the hedonic model requires some strong simplifying assumptions. In particular, the production of products must be convex in amenities, so that any continuous quantity is available, even when conditioned on other quantities. As the industrial organization literature has frequently pointed out, this means there is no space for the creation

---

[1] Griliches (1961) is the classic first reference. See Palmquist (2003) and Freeman (2003, Ch. 11) for introductions in the context of housing prices.

[2] This insight of this tangency condition is often attributed to Rosen (1974), but was first explained by Adelman and Griliches (1961).

of new differentiated commodities. Or, to the extent it is used as an approximation when this condition is not met, the hedonic model implies that any products that do "fill in" the product space have no value (e.g. Trajtenberg 1990). In some cases, this problem has motivated economists to use discrete choice models to estimate quality-adjusted price indices that account for new varieties in lieu of hedonics (Trajtenberg 1990, Nevo 2003).

The crux of the problem caused by non-convexity is the "clumping" of heterogeneous consumer types on a single product. When the choice set is convex and each type continuously available, each household-type will choose it's favorite product-type in the set, which in general will be different from those chosen by other household types. The result is perfect Tiebout sorting in each dimension, where, ceteris paribus, those who prefer more of each attribute will purchase more of it.

When the choice set is not convex, such perfect sorting is not possible. Consider the simplest case where there are more household types than product types, but each type is still available in any quantity. Suppose three household types have constant marginal willingness to pay for air quality of {\$5}, {\$10}, {\$20}, with a mean marginal value of \$11.67. Suppose further that there are two house types (available in sufficient quantity to meet demand) that have air qualities of {0} and {1}. Household 1 will locate in the first house type, household 3 in the second, and household type 2 may locate in either. Rents are maximized by charging a price differential for land of either \$10 and collecting it on households 2 and 3 or \$20 and collecting it on household 3 only. In the former case the price gradient will understate willingness to pay, in the latter case it will overstate it.

Clearly, when there is clumping of heterogeneous households, the determination of which household sets prices is crucial. That determination, in turn, depends on the extent of Tiebout sorting of household by their demands for amenities. Things can get more complicated when the stock of housing of each type is fixed exogenously. Consider another case, now with two amenities (lot size and air quality). There are three households and three houses. The households have willingness to pay for the amenities pairs of {5,5}, {10,5}, and {15,8} (household 3 is the richest). The houses, ordered to match their equilibrium allocation of households, have corresponding amenity quantities {1,1}, {2,1}, and {2,2}. Equilibrium prices are p, p+10, and p+18. Because of the Tiebout sorting, in which discrete house types with the best air quality are chosen by households with the highest demand for it, the price differential for air quality is determined by the household with the highest demand for it, yielding a market value of $8 compared to the true average willingness to pay of $6. Meanwhile, the market value for lot size, at $10, is exactly right.

However, suppose now the houses have amenity pairs {1,1}, {1,2}, {2,1}. Then they will have the same matching to households,[3] but with equilibrium prices p, p+5, and p+12. The market price for air quality, conditioning on lot size, is $5, compared to true average willingness to pay of $6. Now, the household with the highest value for air quality no longer sorts into the house with high air quality. That household also has the highest value for lot size, and prioritizes it over air quality. Consequently, the market value for lot size is now higher than average willingness to pay, at $12 versus $10.

Although such problems are well documented (**citations?**), they are sometimes overlooked in empirical work. For example, Cockburn and Anis (1998) perform a hedonic regression over [small number] rheumatoid arthritis drugs. They find that toxicity has the "wrong" sign in the regression. However, the positive coefficient on toxicity can be explained by the fact that a small proportion of the population only responds to drugs of a certain type, which happen to be toxic. Pakes (2001) suggests that the small market for these drugs is captured by a firm exerting market power, whereas competition in the larger market for the less toxic drugs keeps their prices down. He concludes that, in practice, hedonics often belie the "naïve intuition that [desirable characteristics] should have positive coefficients." Such intuition, he says, "was formalized in a series of early models whose equilibrium implied that the 'marginal willingness to pay equaled the marginal cost of production,'" a model which he characterizes as "very misleading" (p. 13). In fact, however, the problem can be traced, at least in part, to the violation of the convexity assumption of the model. There is instead separation in the drug types, as well as confounding between two amenities, potency (of a type) and toxicity.

[3] Actually, the allocation of households 1 and 2 to houses 1 and 2 is indeterminate.

Recent work in public economics has begun to emphasize the importance of discreteness or discontinuity in exogenous choice sets. To identify values for school quality, Black (1999) uses a matching model to compare houses just on either size of discrete school boundaries. In a more structural approach, Sieg et al. (2003) use a sorting model to recover values when households first sort into discrete communities defined by bundles of public goods, but then conditional on community can endogenously choose houses of continuously available sizes. Others have used more traditional discrete choice models for individual houses (Banzhaf 2002a, Chattopadhay 2000, Palmquist and Israngkura 1999).

Nevertheless, the classic hedonic price regression continues to be the most common approach to recovering marginal values for spatially delineated public goods, with air quality a particularly common application.[4] Perhaps one reason is that most public goods such as air quality, or any amenity defined by distance to a point, appear to be distributed approximately continuously.[5] However, four points of caution need to be made when interpreting exogenous amenities that are seemingly continuous. First, it is not enough that the amenity be distributed over a continuous support; the distribution itself must be continuous, for any atoms in the density of products will lead to clumping of potentially heterogeneous households at that point.[6] Second, even if it can be thought of as being generated from a continuous distribution, any finite sample of houses will not be. Third, even if the *marginal* density of the amenity of interest is continuously distributed, if the joint distribution with other amenities is not then it may cause difficulty for estimating hedonics in practice. Although households should be sorted by their tastes for the continuous amenity after conditioning on the discrete amenities, conventional functional forms and even semi-parametric models may have grave difficulties identifying the price effects. Fourth, and perhaps more practically, amenities such as air pollution, while generated by a continuous process, are often modeled as entering preferences in a discrete form. For example, there are good behavioral reasons to model air pollution with the number of days that a point in space has exceeded a pollution threshold, since such information is often communicated to residents in this form through smog alerts and so on.

[4] See Smith and Huang (1995) for a meta analysis and bibliography, and Banzhaf (2002b), Beron et al. (2000), Chattopadhay (2000), Chay and Greenstone (2000), Zabel and Kiel (1999) for more recent examples.

[5] See, for example, Palmquist (2003), note 25.

Clearly, though these exceedences are generated from a continuous variable (ozone readings at point in space and time), they are not themselves continuous.[7]

The importance of such details about the structure of cities (distribution of amenities) has not previously been explored in depth. This paper uses simulation exercises to gauge their importance for recovering marginal values using the hedonic technique.

The Simulation Model

The simulations in this paper follow an approach suggested by Deck (1987) and Cropper et al. (1988, 1993). The basic idea is to define a set of households with heterogeneous incomes and tastes for housing amenities, define a set of exogenous houses, and find the equilibrium vector of prices that supports an assignment of households to houses. These prices can then form the basis of hedonic regressions. The estimated marginal values from the hedonic function, can then be compared to the true marginal values defined directly from (the known) preference functions. Whereas Cropper et al. (1988) focused on the relative merits of various hedonic functional forms in their comparison, here the experiment is repeated over various city structures to gain insight into their importance in identifying marginal values for amenities.

In the model, N households bid against one another to live in N houses which have already been constructed and are not open to adaptations or additions. The households receive utility from a house h according to its exogenously determined attributes z_h (including both structural attributes like the number of bathrooms and locational attributes like school quality and air quality). They also receive utility from a numeraire commodity y. Thus, formally, they face the following utility-maximization problem:

$$\begin{gathered} \text{Max}_{\{h\}}\ U(z_h, y; a) \quad \ni' \\ p_h + y \le I \end{gathered} \tag{1}$$

---

[6] Moreover, there cannot be an atom at the left end of the support, even though it would be consistent with a continuous distribution.

[7] The problem is inherent in the definition of the commodity, and not a question of measurement. It is distinct from the measurement error that occurs from imputing values from the nearest measurement in space (as in the pollution readings at the nearest monitor) or from reported data that is averaged over space (as in crime rates by jurisdiction).

where I is income and a is a vector of parameters for the utility function. Alternatively, this function can be written as

$$\text{Max}_{\{x\}} \ \text{U}(x'A, I\text{-}x'p; a) \tag{2}$$

where x is an Nx1 vector of binary demands, $x_h \in \{0,1\}$ and $\Sigma x_h = 1$, A is an Nxj matrix of attributes, and p is the Nx1 vector of housing prices.

Define $b_{hi}(u_i)$ as household i's maximum willingness to pay for house h when its utility is u_i. Equilibrium in this market is a vector of prices and of demands that satisfy:

$$p_h^* \ = \ b_{hi}(\text{U}_i\,(\cdot)) \ \text{ if } X_{hi}(p^*, y) = 1 \tag{3a}$$
$$p_h^* \ \geq \ b_{kj}(\text{U}_i\,(\cdot)) \ \text{ if } X_{kj}(p^*, y) = 0 \tag{3b}$$
$$\Sigma_i \text{X}_{\text{hi}} \ = \ 1 \tag{4a}$$
$$\Sigma_h X_{hi} \ = \ 1 \tag{4b}$$

for all i,h. Conditions (3a) and (3b) state that equilibrium rent on a house equals the maximum willingness to pay of its occupant and that no household is willing to pay more for a house that it does not occupy than the household who does occupy it. That is, there are no opportunities for trade. Conditions (4a) and (4b) state that each household must occupy precisely one house, while each house must be occupied by precisely one household.

On the housing side of the market, the simulations use an actual data set of approximately 40,000 Los Angeles houses that were on the market in 1990, a data set previously described in Sieg et al. (2003) and Banzhaf (2002a,b). These data were used in order to have a realistic city structure. They are described in more detail in the following section. Each simulation samples N=1000 houses from this larger set.

The household side of the market requires the specification of preference functions. For these preliminary results, utility is assumed to be a Cobb-Douglas in the numeraire good and continuous housing attributes:

$$\text{U}_\text{i}(y, z) \ = \ \ln(y) + \Sigma_j \, a_{ij}\cdot\ln(z_\text{j}) + a_{id}D_d \tag{6}$$

where D_d is the dth attribute defined as an indicator variable (locational fixed effects or presence of a swimming pool, e.g.). The utility parameters a are random across individuals, and normalized to the income parameter. For realism, these parameters were estimated from a discrete choice model described in Banzhaf (2002a). Table 1 shows the parameters of the distributions. As indicated in the table, some parameters are free to be positive or negative, some are censored at slightly positive (1% of the mean value), while educational variables are censored at zero. The column "value share" gives a rough gauge of the importance of each variable; it is the marginal value of the amenity times the quantity, divided by the sum of this product over amenities (i.e., for amenity k it is $MWTP_k*z_k/\Sigma_j MWTP_j*z_j$), all averaged over houses/households. As can be seen, educational amenities are very important, while air quality is less so, and public safety, bathrooms, and lot size still less.

In addition to the utility functions, each household is endowed with an initial income. Total household incomes were randomly drawn from a censored log-normal income distribution estimated from the 1990 census for Los Angeles area home owners. The parameters of the distribution are μ=11.1 and σ^2=0.4 and is censored on ($35k, $130k). The median household income is thus $66,200.

In principle, the economic equilibrium described in equations (3) and (4) can be modeled as a mixed complementarity problem. However, characterizing the problem in this way requires solving N^2+2N equations, which is not tractable for very large N (for $N=1000$ this is 1,002,000 equations!). Instead, the problem was characterized as a linear program, requiring the solution of only $2N+1$ equations, which was iterated to converge to an economic equilibrium. The remainder of this section describes this problem in some detail; disinterested readers may skip ahead to the following section.

The basic starting point is the assignment of N households to N houses. The assignment problem is to choose an NxN permutation matrix P (with precisely one 1 in each row and each column and zeros in every other element) to maximize

$$\mathrm{Max}_{\{P\}}\ 1'_{1\mathrm{x}N}[b_{hi}]_{N\mathrm{x}N}P_{N\mathrm{x}N}1_{N\mathrm{x}1}$$

There are N! choices of P, which even for a small sample becomes very large. This makes the problem all but intractable. However, Koopmans and Beckman (1957) showed that the linear assignment problem can be turned into a linear programming problem.

First, begin with some level of utility taken as a parameter. At some starting value of $u_i(\ )$ find the b_{hi} (each household's bid for each house). Then solve the following maximization problem:

$$\begin{aligned} &\mathrm{Max}_{\{X\}}\ \Sigma_h \Sigma_i\ b_{hi} X_{hi} \quad \ni' \\ &\Sigma_h\ X_{hi} = 1 \quad \forall\ i \\ &\Sigma_i\ X_{hi} = 1 \quad \forall\ h \\ &X_{hi} \geq 0 \quad \forall\ h,i \end{aligned} \tag{7}$$

Here, X_{hi} is interpreted as the "share" of house h allocated to household i. Unlike the demand vector in (2)-(4), in principle X_{hi} may vary freely and is not constrained to 0 or 1. Hence, a very large linear assignment problem is turned into a linear programming problem. But as Koopmans and Beckman note, the constraints in (7) do restrict X to a convex polyhedron, and a linear function defined over this region will find its maximum at a vertex. Hence, the optimal X_{hi} will in fact be 0 or 1.

While this procedure gives a solution to the linear assignment problem, at this point prices have not yet entered the picture, and we have done nothing to ensure conditions (3). That is, the solution may not be Pareto optimal. To see how we can ensure this condition, first substitute (3a) into (3b):

$$b_{hi} \geq b_{ki} \text{ if } X_{hi}(p^*, y) = 1 \text{ and if } X_{ki}(p^*, y) = 0 \tag{8}$$

Wheaton (1974) identifies the duel problem to (7) as

$$\mathrm{Min}_{\{r,\ v\}}\ \Sigma_h\ r_h + \Sigma_i\ v_i \quad \ni' \quad p_h + v_i \geq b_{hi} \tag{9}$$

where v_i is the side payment to household i needed to maintain utility levels and r_h is the rental rate for the property. By the rules of complementary slackness, we know $r_h + v_i = b_{hi}$ if (h,i) is in basis and $r_h + v_k \geq b_{hk}$ if (h,k) is not. It is easy to see that a sufficient condition to guarantee (8) is that $v_i = v_k$, since then, substituting $(v_i = b_{hi} - p_h)$ for v_k in $(p_h + v_k \geq b_{hk})$, we have $b_{hi} \geq b_{hk}$. Finally, to satisfy (3a), we need only set $v = 0$, so that the rental rate is equal to the willingness to pay. This is possible since one of the $2n$ constraints is redundant, making the shadow values of p and v unique only up to an additive constant.

To obtain the sufficient condition $v_i = v_k$, Wheaton suggests the following iterative process. First calculate (7) and (9) as above, with the b_{hi} calculated from some reference utility. Then normalize the v_i so that they are all negative. For the next round find the new level of utility for each household when they are living in the house assigned to it in the previous round and paying a price equal to its bid minus the side payment v_i (which is negative). That is, for all households i, at round t+1 find u^{t+1} to solve $b_{h(t)i}(u^{t+1}) = r_{h(t)}$ where $h(t)$ is the house assigned to i in round t. Then in round t+1 recalculate the b_{hi} matrix and again solve (7) and (9). This process is repeated until t=t* such that $v_i \approx 0 \; \forall \; i$ at (9). This approach iteratively raises bids until convergence is reached, although global convergence is not guaranteed at bids less than income or even at all. The routine was implemented using GAMS's BDMLP routine for linear programming.

Description of Data and Baseline City Structure

As noted above, the housing stock used in these simulations is from a sample of homes sold in Los Angeles in 1990. The amenities include fixed effects for county (Orange, San Bernardino, Riverside, and Ventura relative to Los Angeles); an indicator for proximity to the coast; neighborhood amenities for ozone (measured as days with an ozone exceedence), teacher-student ratios, achievement test scores, and public safety (renormalized from the crime rate); and physical amenities for the number of bathrooms, the size of the building, the size of the lot, the presence of a fireplace, the presence of a swimming pool, and the age of the house. In the reference case, ozone exceedences are imputed to each home as the value at the nearest air quality monitor. In a comparison case, it is imputed as a weighted average of the three nearest monitors, which gives rise to a more continuous distribution of air quality. Table 2 gives descriptive statistics for the data.

Unlike empirical work, simulation exercises such as this raise no questions about data quality, since they are taken as true in the model. However, understanding these data is important for understanding how households may sort in the urban structure of the model. Table 3 gives a first cut at a deeper understanding of the data by showing the table of correlation coefficients for all amenities. It also gives the R^2 from a reverse regression of each amenity on all others to gauge the independence of each amenity. In general, multicolinearity does not appear to be a severe problem, as none of the reverse regressions yield very high R^2, at least not in comparison to the R^2s of the hedonic regressions (see below). However, there are distinct patterns to the data. The table shows that bathrooms and buildings are the least independent due to their high correlation with one another (0.75). Age, the two measures of ozone, and public safety are next, with R^2s in the range of 0.45 to 0.58. All three of these variables are correlated with other spatial amenities, and age is also correlated with measures of building size.

To go further into the discreteness and clustering of the data, Figures 1 through 3 give three plots of air quality, bathrooms, and building size respectively plotted against the teachers-per-student variable. The sets of houses at each teacher level, defined by each school district, are unmistakable, as are the sets of air quality values defined by each monitor and the bathrooms defined in quarters. Building size, however, seems to be available fairly continuously in each school district. The extent of Tiebout sorting permitted with such data, and its implications for hedonic regressions, is explored in the following section.

Results of Simulations and Hedonic Regressions.

The simulations described above are performed with the housing stock as described above (the reference case), as well as for other scenarios designed to test hypotheses about the importance of urban structure. For now, each scenario is repeated five times (additional simulations will be added in future drafts).

To confirm the success of the simulations—and the importance of patterns of Tiebout sorting—an initial scenario was implemented with the baseline city structure and with complete homogeneity of households (identical tastes and incomes). In this case, there is no question of clustering as the price of each house will differ from its neighbors according to the exact willingness to pay of all households. In other words, the prices trace out the preference ordering. The results of these simulations are summarized in Table 5. The Table shows the average (over the simulations) of the percentage error in the estimated mean marginal willingness to pay relative to the true mean willingness to pay, for each amenity and for each of several specifications. The specifications are the common log-linear form; a linear-log form; translog I, which omits interactions with dummy variables; translog II, which includes all possible interactions; a Box-Cox regression, and a semi-parametric regression.[8]

As the table shows, the fit is good and the errors are generally very small in this homogenous case, especially in the log linear functional specification, as one would expect. Public safety is one of the hardest to pin down, but the percentage errors are on top of small true values, so the absolute errors are actually fairly small. Age is the most sensitive to functional form, and is estimated correctly only in the linear-log specification. The semi-parametric regression performs badly here as it does in all the experiments to follow. The problem with the non-parametric approach may be in the inability in these simulations to tailor the bandwidth to the data for each amenity-simulation combination. Inspections of the non-parametric regressions suggest they fit the data well, but when under-smoothed oscillate around the trend in the data, generating negative marginal values alternated with very high values.

As shown in Table 6, when heterogeneity is introduced into tastes and incomes the good fit continues for most models but matters get much worse for estimating marginal values. Mean marginal values for building size and lot size, the most continuous of the variables, continue to be predicted well in the translog forms. However, the errors in mean marginal values are much worse for the other variables, and often greater than 25%. While errors of 25% or even more might be acceptable for much empirical economic work, it should be remembered that these errors occur with perfectly clean data, in specifications with absolutely no omitted variables, and in a context where we have assured that all households are in equilibrium. Such large errors seem more surprising in such ideal laboratory conditions and can only raise questions about the use of hedonic regressions in practice.

A review of the results in Cropper et al. (1988) shows that they too found large errors in marginal values for many amenities, even before introducing measurement error or omitted

[8] The semiparametric regression uses a semilog model to parametrically difference out all amenities other than the amenity of interest in each row. The procedure involves higher-order differencing described in Yatchew (1998), followed by locally linear regressions for the variable of interest. Given the vast number of regressions run in the course of the many simulations, exploring sensitivity to bandwidth on a case-by-case basis was not possible. Instead, the "optimal" bandwidth was used in all cases: $0.9*(N^{-5})*\min\{\sigma, IQR/1.34\}$, where IQR is the

variables into their hedonic simulations. While they focus on the relative merits of various functional forms, the question posed here is what are the properties of the urban structure and resulting equilibria that drive such results.

Figure 4a shows the extent of Tiebout sorting on a variable like building size, which is fairly continuous in the sample and whose marginal values are estimated well in the hedonic regressions. The figure shows the matching of households, indicated by their taste for building size, to the size of the buildings they purchase. The figure shows an unmistakable pattern of Tiebout sorting, with households who desire large homes outbidding others to live in them. However, tastes are not the only factor at work. Ceteris paribus, wealthier households are also more likely to outbid other households for larger homes. Accordingly, there is similar evidence of matching by wealth and building size. To combine both effects, Figure 4b shows Tiebout sorting with households indicated by their marginal willingness to pay *at a fixed point* (the mean of the data). (The fixed point is required since otherwise the effect of diminishing marginal willingness to pay would create a downward sloping line.) Figure 4b indicates even stronger evidence of Tiebout sorting by building size.[9]

Figure 5 shows the extent of Tiebout sorting, again represented by willingness to pay at a point, for the number of teachers. Tiebout sorting is still clearly present, but there is also unmistakable clustering of heterogeneous households in the same school districts. This clustering may account for the larger errors in estimating marginal willingness to pay for teacher ratios than for building size.

Figures 6 and 7 repeat the comparison for air quality and bathrooms respectively. In both cases there is strong evidence of clustering on the discrete values and indeed little visual evidence of sorting by demand for the amenities at all. Again, this pattern of sorting may account for the even poorer performance in the hedonic regressions at predicting marginal willingness to pay for these amenities. The pattern may result from a combination of two

inter-quartile range. Marginal values were computed as the derivative of the kernel estimator with respect to the variable of interest.

[9] While incorporating information about income creates a tighter graph in this example, it should be noted here that this is not always the case. While, holding income constant, households should always sort by tastes, holding tastes constant households might not always be able to sort by income for all amenities. In some cases,

factors. Unlike building size, the distribution of the data is fairly discrete, as discussed previously. But relative to an amenity like the teacher-student ratio, which also comes in discrete units, these amenities are just not a priority to households. Consequently, the households sort on other amenities and take air quality and bathrooms as something of a residual.

Table 4 provides a non-graphical summary of such issues for each amenity. For the reference case, it shows the simple correlation between each amenity and the marginal willingness to pay of households at a fixed point, their taste parameters only, and their incomes only. As shown in the table, correlations are high for building size and teacher ratios but much lower for other variables and even negative for some locational indicator variables. These simple correlations may mask stronger correlations once other amenities are fixed. That is, heuristically, if households sort first into school districts, and if school quality is negatively correlated with air quality, we might not find a strong correlation between tastes and air quality, but we might still find such correlation within each school district. In fact, there is little evidence for sorting on air quality even when conditioning on each school-district/bathroom cell.

Two experiments relax the structure of this urban environment in *one* amenity that is often of empirical interest, namely, air quality. First, the measure of air quality in the reference scenario, the number of exceedences of the ozone standard at the nearest air quality monitor, is replaced by a distance-weighted average of the three nearest monitors, thereby smoothing the differences among homes and making the distribution more continuous. The resulting distribution has approximately the same mean and is over a slightly narrower range. The simulated market was repeated with these data, along with the hedonic regressions and estimated marginal values. The results of this experiment are shown in Table 7. Surprisingly, the errors in the estimated mean willingness to pay for air quality are now even *larger*. Although this measure of air quality is more continuous, as shown in Table 3 it is also more correlated with other amenities. This correlation may make it harder to sort on air quality. A second experiment takes this set of air quality values and *randomly* distributes them to houses

households may use their income to purchase one amenity, which if negatively correlated with another would diminish the extent of Tiebout sorting in the second.

defined by their other amenities. In this experiment, the relationship between air quality and space is completely severed. As shown in Table 8, the error in mean marginal willingness to pay is now greatly reduced, and indeed less than 3 percent in all cases except the semi-parametric model.

A final pair of experiments replicate this same logic for all variables at the same time, though this time in reverse order. First, while maintaining the same individual marginal distributions, all the amenities are randomly matched together. That is, there is no longer any correlation among amenities. The results of this experiment are summarized in Table 9. The table shows that the mean marginal willingness to pay is now better estimated for every amenity, certainly for the translog specifications and almost for every cell in the table. Further, Table 4 shows that sorting is now stronger for every amenity with the exception of fireplace and swimming pool. In some cases, the improvement is very large: figures in boldface indicate changes of 0.05 or more. Next, the amenities levels are randomly "spread out" from their previous values in such a way as to be continuous. In this imaginary city structure, it is possible to be x% coastal, y% Riversidish, and z% San Bernardinish, to have any quantity of bathrooms on the real line, and so on.[10] As shown in Table 10, the results of this further experiment are more mixed, with some estimated values improving and some getting worse. Interestingly however, the estimated marginal values for bathrooms, the most discrete of the variables, does improve dramatically.

Discussion and Conclusion

Clearly, the two pairs of experiments together suggest that the spatial structure of the data is more important than discreteness, at least in the case of air quality. Two possible *econometric* factors bear consideration here. First, any problem of estimating the parameters due to multicolinearity is overcome when the data are randomly distributed. Second, any correlation in the error structure is also overcome. Such correlation has been of increasing interest in hedonic models (e.g. Bell and Bockstael 2000). However, to the extent it is an empirical issue related to spatially distributed measurement error or unobservables, this explanation can be

[10] It is not a mean-preserving spread, but rather maintains the range of the original data (e.g., indicator variables are no longer only zero or one, but are still bounded by zero and one).

ruled out for these controlled simulations.[11] Furthermore, both of these econometric issues relate to the efficiency in the estimation, but the results of the reference case (Table 6) suggest bias is a problem [although, at this point, 5 repetitions is not really sufficient to say so with confidence].

A more plausible explanation lies in the *economic* structure of the problem. As stressed previously households do not sort on air quality in any discernable way, even when conditioning on large cells. Table 4 provides weak evidence that households better sort on air quality when it is divorced from other amenities. The correlation between air quality and the corresponding taste parameter increases from 0.289 to 0.371 in this experiment, although correlation with income actual declines. Because air quality is apparently a low priority, households heuristically sort based on other amenities, and then sort on air quality based on what is available at that point. The result is great variation in willingness to pay along the air quality line. To some extent, this is partly a question of more flexible functional forms. But viewed another way it is more radical. Interactions can account for linear trends with other amenities, but not discontinuous relationships. There may not be any sense in which a finite set of houses can generate a smooth function of marginal values on air quality when there is this kind of dependence. Over a narrow range price differentials must be tied down by a set of households with potentially very different demands, a kind of clustering generalized from a single value to a neighborhood.

The emphasis here is on *marginal* values. As noted previously, price levels are predicted very well, with large R^2s and small mean square errors. To some extent, this emphasizes old concerns about evaluating models purposed for welfare measurement based on criteria of fit (Smith 1990). It also lends support to consideration of traditional logit models and newer discrete or discrete-continuous sorting models as an alternative.

---

[11] Likewise, there is no structural reason for any spatial lagged dependence, since the prices are determined independently.

References

Adelman, Irma, and Zvi Griliches. 1961. "On an Index of Quality Change." *Journal of the American Statistical Association* 56: 535-548.

Banzhaf, H. Spencer. 2002a. "Quality Adjustment for Spatially-Delineated Public Goods: Theory and Application to Cost-of-Living Indices in LA," RFF Discussion Paper 02-10. Washington, DC: Resources for the Future.

Banzhaf, H. Spencer. 2002b. "Green Price Indices," RFF Discussion Paper 02-09. Washington, DC: Resources for the Future.

Bartik, Timothy J. 1988. "Measuring the Benefits of Amenity Improvements in Hedonic Price Models." *Land Economics* 64(2): 172-183.

Bell, Kathleen P., and Nancy E. Bockstael. 2000. "Applying the Generalized-Moments Estimation Approach to Spatial Problems Involving Microlevel Data." *Review of Economics and Statistics* 82: 72-82.

Beron et al. 2000.

Chattopadhyay, Sudip. 2000. "The Effectiveness of McFaddens's [*sic*] Nested Logit Model in Valuing Amenity Improvements." *Regional Science and Urban Economics* 30: 23-43.

Chay, Kenneth Y., and Michael Greenstone. 2000. "Does Air Quality Matter? Evidence from the Housing Market." Mimeo. Berkeley, CA.

Cockburn and Anis (1998) NBER working paper.

Cropper, Maureen L., Leland B. Deck, and Kenneth E. McConnell. 1988. "On the Choice of Functional Form for Hedonic Price Functions." *Review of Economics and Statistics* 70(4): 668-675.

Cropper, Maureen L., Leland Deck, Nalin Kishor, and Kenneth McConnell. 1993. "Valuing Product Attributes Using Single Market Data: A Comparison of Hedonic and Discrete Choice Approaches." *Review of Economics and Statistics* 75(2): 225-232.

Deck, Leland B. 1987. *The Estimation of Consumer Preferences in Hedonic Markets: A Simulation Test.* Dissertation, University of Maryland.

Freeman, A. Myrick. 1974. "On Estimating Air Pollution Control Benefits from Land Value Studies." *Journal of Environmental Economics and Management* 1: 74-83.

Freeman, A. Myrick. 2003. *The Measurement of Environmental and Resource Values: Theory and Methods*, 2nd ed. Washington, DC: Resources for the Future.

Griliches, Zvi. 1961. "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change." In *The Price Statistics of the Federal Government*, by George Stigler et al. National Bureau of Economic Research.

Kanemoto, Yoshitsugu. 1988. "Hedonic Prices and the Benefits of Public Projects." *Econometrica* 56(4): 981-989.

Koopmans, Tjalling C., and Martin Beckman. 1957. "Assignment Problems and the Location of Economic Activities." *Econometrica* 25(1): 53-76.

Nevo, Aviv, 2003, "New Products, Quality Changes, and Welfare Measures Computed from Estimated Demand Systems," *Review of Economics and Statistics* 85(2): 266-75.

Pakes, Ariel. 2001. "A Reconsideration of Hedonic Price Indices, with an Application to PCs." Mimeo. Harvard University.

Palmquist, Raymond B. 2003. "Property Value Models." In *Handbook of Environmental Economics*, vol. 2, ed. by Karl-Gören Mäler and Jeffrey Vincent. Amsterdam: North-Holland, forthcoming.

Palmquist, Raymond B., and Adis Israngkura. 1999. "Valuing Air Quality with Hedonic and Discrete Choice Models." *American Journal of Agricultural Economics* 81: 1128-1133.

Rosen, Sherwin. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." *Journal of Political Economy* 82:34-55.

Sieg, Holger, V. Kerry Smith, H. Spencer Banzhaf, and Randall Walsh. "Estimating the General Equilibrium Benefits of Large Changes in Spatially Delineated Public Goods." *International Economic Review*, forthcoming.

Smith, V. Kerry. 1990. "Estimating Recreation Demand Using the Properties of Implied Consumer Surplus." *Land Economics* 66: 111-120.

Trajtenberg, Manuel. 1990. *Economic Analysis of Product Innovation: The Case of CT Scanners*. Cambridge, MA: Harvard University Press.

Wheaton, William C. 1974. "Linear Programming and Locational Equilibrium: The Herbert-Stevens Model Revisited." *Journal of Urban Economics* 1(3): 278-287.

Yatchew, Adonis. 1998. "Nonparametric Regression Techniques in Economics." *Journal of Economic Literature* 36: 669-721.

Zabel, Jeffrey E., and Katherine A. Kiel. 2000. "Estimating the Demand for Clean Air in Four United States Cities." *Land Economics* 76(2): 174-194.

Table 1. Utility Parameters for Each Variable

	Mean	Std Dev	Censored?	Value Share
Orange Co	-0.177	0.017	No	-5.4%
Riverside Co	-0.374	0.037	No	-11.4%
San Bernardino	-0.354	0.35	No	-10.8%
Ventura Co	-0.074	0.0074	No	-2.3%
Coast	0.520	0.052	Zero	13.0%
Days w/o Ozone Alert	0.405	0.041	1 percent	12.3%
Teachers per student	1.309	4.84	Zero	35.0%
Test Score	0.914	0.085	Zero	27.9%
Public Safety	0.172	0.349	1 percent	6.9%
Bathrooms	0.056	0.046	1 percent	1.8%
Building size (sq ft)	0.511	0.427	1 percent	15.0%
Lot size (sq ft)	0.107	0.0162	1 percent	3.3%
Fire place	0.077	0.0077	Zero	2.3%
Pool	0.078	0.0078	Zero	2.4%
Age	-0.0059	00321	No	-0.2%

Table 2. Descriptive Statistics of Underlying Housing Data

	Mean	Std Dev	Min	Max
Orange Co	0.1280	--	0	1
Riverside Co	0.1783	--	0	1
San Bernardino	0.1227	--	0	1
Ventura Co	0.0693	--	0	1
Coast	0.0116	--	0	1
Days w/o Ozone Alert	315	38	241	364
Teachers per student	4.052	0.174	3.509	5.780
Test Score	4.891	0.511	3.78	6.78
Public Safety	2,387	157	412	2,852
Bathrooms	2.005	0.666	0.5	6
Building size (sq ft)	1588	535	504	4934
Lot size (sq ft)	8,383	6,715	540	99,752
Fire place	0.6034	--	0	1
Pool	0.1475	--	0	1
Age	26.12	20.20	1	94

Based on 39,347 housing in the Los Angeles area.

Table 3. Correlation and Multiple Correlation Among Amenities

	R^2	Ornge	Rivrsd	San Bern	Ventra	Coast	Ozone (Nbr)	Ozone (wt)	Teach	Test	Safety	Baths	Buildg	Lot	Fire place	Pool	Age
Orange Co	0.43	1.00															
Riverside Co	0.42	--	1.00														
San Bernardino	0.24	--	--	1.00													
Ventura Co	0.30	--	--	--	1.00												
Coast	0.03	0.09	-0.05	-0.04	0.03	1.00											
Days w/o Ozone Alert (nbr)	0.50	0.37	-0.27	-0.48	0.28	0.12	1.00										
Days w/o Ozone Alert (wt)	0.58	0.39	-0.31	-0.52	0.29	0.13	(0.95)	1.00									
Teachers per student	0.08	0.10	-0.10	-0.07	-0.05	0.08	0.18	0.19	1.00								
Test Score	0.38	0.53	-0.12	-0.04	0.06	0.09	0.16	0.16	0.06	1.00							
Public Safety	0.45	0.08	-0.42	-0.08	0.42	0.01	0.19	0.21	0.10	0.30	1.00						
Bathrooms	0.66	0.05	0.07	0.10	0.09	-0.04	-0.12	-0.13	-0.02	0.18	0.19	1.00					
Building size (sq ft)	0.63	0.05	0.05	0.04	0.11	-0.06	-0.06	-0.07	0.00	0.18	0.22	0.75	1.00				
Lot size (sq ft)	0.15	-0.09	0.17	0.03	0.03	-0.07	-0.17	-0.18	-0.08	0.01	0.02	0.13	0.27	1.00			
Fireplace	0.27	-0.31	0.26	0.18	0.10	-0.06	-0.26	-0.28	-0.02	-0.11	-0.04	0.25	0.27	0.11	1.00		
Pool	0.08	0.01	-0.01	0.00	-0.01	-0.04	-0.07	-0.06	0.01	0.04	0.07	0.19	0.24	0.13	0.11	1.00	
Age	0.58	-0.03	-0.40	-0.23	-0.12	0.07	0.27	0.30	0.14	-0.17	-0.06	-0.56	-0.41	-0.11	-0.25	-0.07	1.00

R^2 is the coefficient of determination in a regression of each amenity on all others. (Ozone-nbr is used on the right-hand side for non-ozone variables.)

Table 4. Index of Tiebout Sorting by Amenity

	Reference Case			Air Quality Independent, Continuous		All Independent		All Independent, Continuous	
	MWTP at Mean	Taste Parameter	Income	MWTP at Mean	Taste Parameter	MWTP at Meanr	Taste Parameter	MWTP at Mean	Taste Parameter
Orange Co	-0.028	0.293	0.123	**0.028**	0.280	**0.193**	**0.338**	**0.123**	0.313
Riverside Co	0.413	0.328	-0.353	0.405	0.349	0.431	**0.415**	0.440	0.444
San Bernardino	0.354	0.311	-0.298	0.319	0.363	0.341	0.373	**0.417**	**0.401**
Ventura Co	-0.147	0.120	0.200	-0.098	0.127	0.040	**0.191**	0.011	**0.124**
Coast	0.279	0.105	0.234	0.269	0.119	0.272	0.141	**0.498**	**0.426**
Days w/o Ozone Alert	0.413	0.289	0.346	**0.330**	**0.371**	0.272	0.322	0.302	**0.227**
Teachers per student	0.768	0.753	0.187	0.768	0.755	0.776	0.775	**0.841**	**0.838**
Test Score	0.553	0.349	0.469	0.523	0.362	0.518	**0.414**	0.463	0.423
Public Safety	0.613	0.537	0.431	0.608	0.538	0.588	**0.620**	**0.686**	**0.705**
Bathrooms	0.497	0.407	0.332	0.501	0.404	**0.593**	**0.652**	0.557	**0.586**
Building size (sq ft)	0.935	0.800	0.436	0.935	0.795	0.936	**0.855**	0.898	0.865
Lot size (sq ft)	0.317	0.324	0.188	0.342	0.325	0.338	0.335	**0.420**	**0.471**
Fire place	0.198	0.258	0.128	0.222	0.276	0.236	0.266	**0.105**	**0.178**
Pool	0.348	0.223	0.289	0.360	0.237	**0.185**	0.216	0.180	**0.156**
Age	0.654	0.701	0.011	0.661	0.707	**0.753**	**0.778**	**0.661**	**0.687**

Bold figures highlight movements of +/-0.050 or more in corresponding figure from previous model.

Table 5. Percentage Errors in Estimated Mean Willingness to Pay (Homogenous Households)

	Specification					
Amenity	**Log-linear**	**Linear-Log**	**Trans log I**	**Trans log II**	**Box-Cox**	**Semi-Parametric**
Air Quality	-5.4%	1.1%	3.1%	3.7%	1.4%	-43.9%
Teachers/Student	1.6%	0.2%	-0.4%	-0.4%	1.6%	-30.8%
Test Score	-7.4%	-2.7%	-1.7%	-0.5%	-6.3%	-23.1%
Public Safety	94.6%	16.4%	15.8%	-6.7%	52.3%	30.0%
Bathrooms	23.1%	-3.4%	-16.2%	-3.3%	5.2%	-77.0%
Building Size	-20.3%	-2.7%	1.2%	-0.3%	-9.7%	-1.8%
Lot Size	-65.6%	-3.0%	-0.9%	0.0%	-25.3%	-8.4%
Age	-57.1%	4.7%	132.3%	67.9%	-40.7%	-47.1%
R^2	0.943	0.998	0.992	0.998	--	--

Table 6. Percentage Errors in Estimated Mean Willingness to Pay (Reference Case)

	Specification					
Amenity	**Log-linear**	**Linear-Log**	**Trans log I**	**Trans log II**	**Box-Cox**	**Semi-Parametric**
Air Quality	-26.4%	-14.4%	-6.4%	-25.6%	-8.6%	-22.6%
Teachers/Student	-30.5%	-21.9%	-11.7%	-12.4%	-30.9%	-57.8%
Test Score	13.5%	23.9%	13.0%	4.8%	12.7%	-25.4%
Public Safety	-105.0%	-166.0%	-49.7%	-36.8%	-114.1%	-97.1%
Bathrooms	-34.9%	-94.3%	-14.4%	-21.2%	-83.9%	-76.1%
Building Size	24.9%	41.9%	4.1%	3.8%	24.6%	6.0%
Lot Size	-57.5%	36.9%	1.5%	-1.0%	25.5%	-12.0%
Age	-95.9%	-36.3%	-21.9%	-7.7%	-29.9%	-89.6%
R^2	0.937	0.863	0.988	0.991	--	--

Table 7. Percentage Errors in Estimated Mean Willingness to Pay (Continuous Air Quality)

	Specification					
Amenity	Log-linear	Linear-Log	Trans log I	Trans log II	Box-Cox	Semi-Parametric
Air Quality	-34.3%	-15.8%	-11.8%	-35.2%	-9.4%	-38.5%
Teachers/Student	-30.0%	-21.4%	-10.9%	-12.1%	-30.6%	-58.0%
Test Score	13.7%	24.3%	13.5%	6.1%	12.9%	-26.2%
Public Safety	-106.9%	-165.9%	-47.2%	-39.5%	-114.0%	-99.1%
Bathrooms	-35.7%	-94.8%	-14.7%	-21.6%	-83.9%	-75.4%
Building Size	25.2%	42.0%	4.1%	4.0%	24.9%	5.8%
Lot Size	-57.5%	37.3%	0.9%	-1.2%	25.6%	-13.9%
Age	-96.1%	-36.6%	-22.0%	-8.6%	-29.9%	-90.3%
R^2	0.937	0.862	0.988	0.991	--	--

Table 8. Percentage Errors in Estimated Mean Willingness to Pay (Independent, Randomly Distributed Air Quality)

	Specification					
Amenity	Log-linear	Linear-Log	Trans log I	Trans log II	Box-Cox	Semi-Parametric
Air Quality	-1.3%	3.0%	-2.8%	-1.4%	-2.0%	-21.8%
Teachers/Student	-31.5%	-22.4%	-12.5%	-14.8%	-30.8%	-59.1%
Test Score	13.2%	23.6%	14.4%	6.6%	12.1%	-22.7%
Public Safety	-96.5%	-156.3%	-44.8%	-38.9%	-109.5%	-82.5%
Bathrooms	-32.0%	-94.7%	-12.6%	-17.0%	-82.1%	-74.7%
Building Size	23.7%	40.2%	3.2%	2.5%	23.1%	4.7%
Lot Size	-55.7%	42.6%	3.6%	3.9%	27.8%	-8.8%
Age	-95.2%	-32.2%	-18.2%	-8.4%	-27.9%	-88.4%
R^2	0.933	0.857	0.987	0.994	--	--

Table 9. Percentage Errors in Estimated Mean Willingness to Pay (Independent, Randomly Distributed Variables)

	Specification					
Amenity	**Log-linear**	**Linear-Log**	**Trans log I**	**Trans log II**	**Box-Cox**	**Semi-Parametric**
Air Quality	-5.9%	-5.7%	-3.7%	-2.8%	-7.5%	-28.5%
Teachers/Student	-24.7%	-22.8%	-4.1%	-4.7%	-28.6%	-57.1%
Test Score	-3.5%	4.4%	-3.3%	-1.9%	-0.1%	-28.0%
Public Safety	-14.6%	-22.3%	-1.0%	5.5%	-23.5%	-10.5%
Bathrooms	-10.6%	-20.0%	-10.6%	-11.2%	-18.0%	-83.6%
Building Size	13.3%	27.1%	-1.0%	-1.5%	17.8%	-6.2%
Lot Size	-51.3%	19.9%	0.2%	3.2%	8.6%	-8.1%
Age	-90.8%	-54.6%	2.0%	2.4%	-61.8%	-88.9%
R^2	0.881	0.796	0.973	0.979	--	--

Table 10. Percentage Errors in Estimated Mean Willingness to Pay (Independent, Randomly Distributed Continuous Variables)

	Specification					
Amenity	**Log-linear**	**Linear-Log**	**Trans log I**	**Trans log II**	**Box-Cox**	**Semi-Parametric**
Air Quality	-2.26%	0.91%	-5.68%	-3.37%	-1.95%	-7.09%
Teachers/Student	-15.59%	-12.49%	-12.22%	-13.23%	-16.97%	-9.53%
Test Score	4.14%	7.21%	1.33%	0.57%	4.13%	-2.65%
Public Safety	-19.15%	-23.91%	-11.62%	-10.55%	-22.25%	-19.66%
Bathrooms	2.78%	5.49%	4.75%	1.39%	0.12%	-4.40%
Building Size	-1.83%	-4.37%	-7.35%	-7.28%	-8.02%	-8.20%
Lot Size	-9.96%	8.09%	-0.66%	-0.15%	2.95%	-15.30%
Age	-60.44%	-1.62%	12.75%	10.40%	-14.74%	-42.22%
R^2	0.904	0.831	0.976	0.983	--	--

Figure 1. Air Quality versus teacher-student ratio (reference scenario).

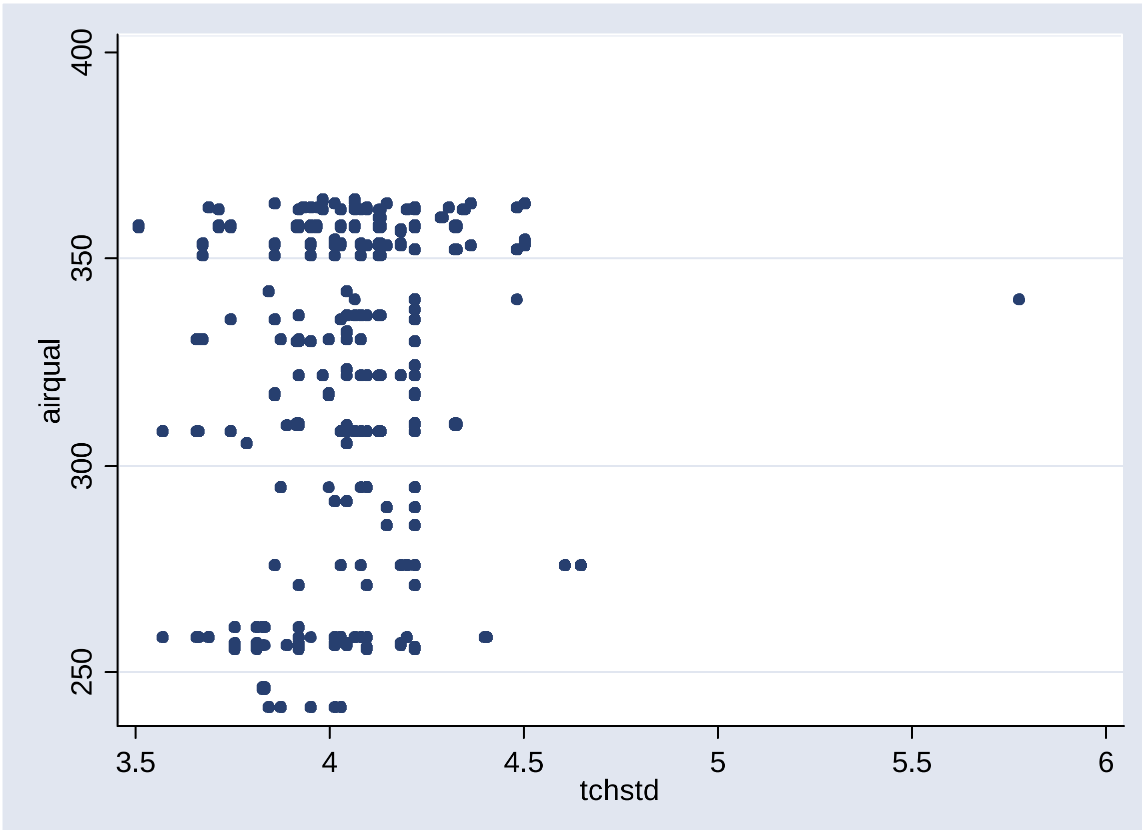

Figure 2. Bathrooms versus teacher-student ratio (reference scenario).

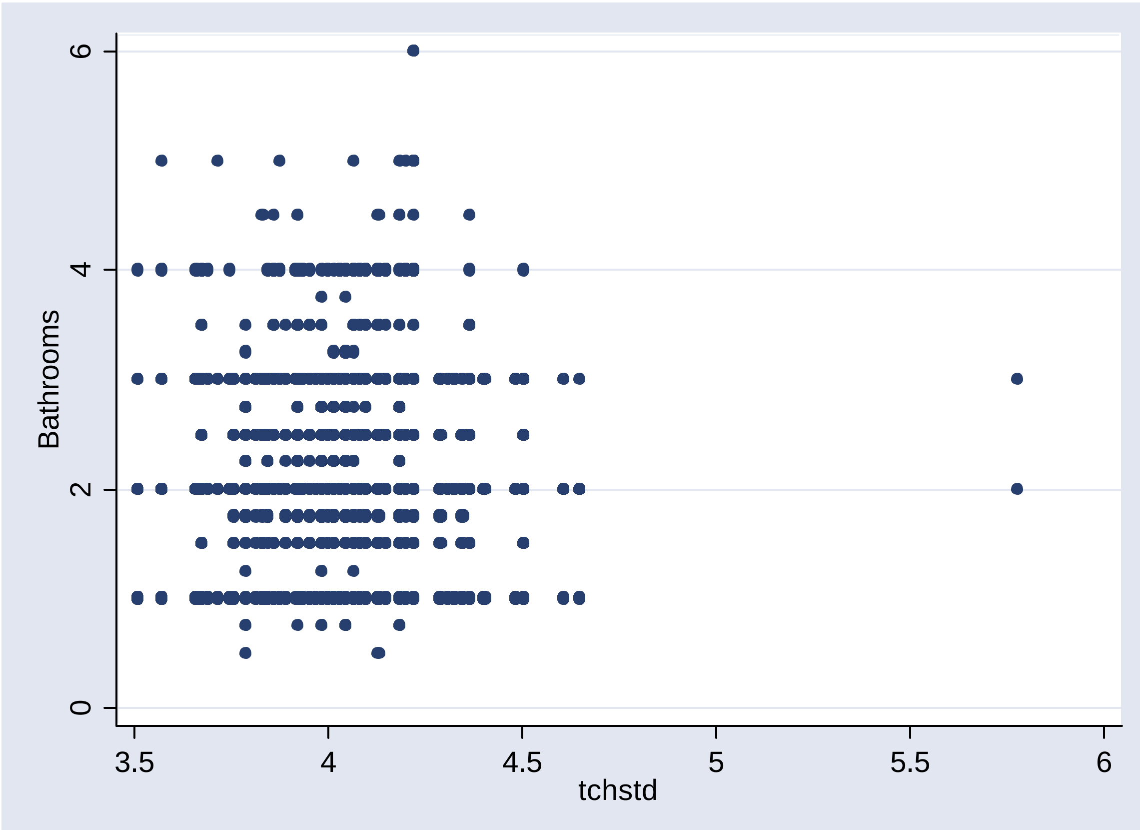

Figure 3. Building size versus teacher-student ratio (reference scenario).

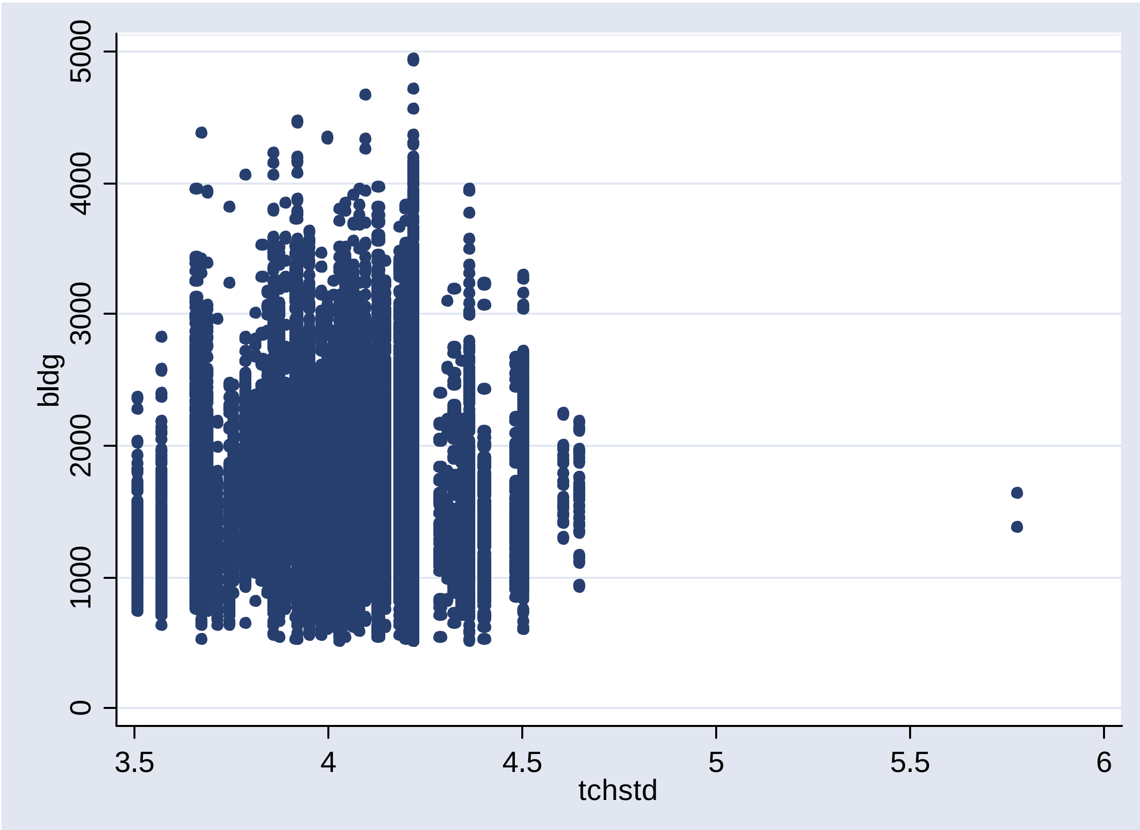

Figure 4a. Sorting on Building Size: Taste Parameter Matched to Amenity Levels

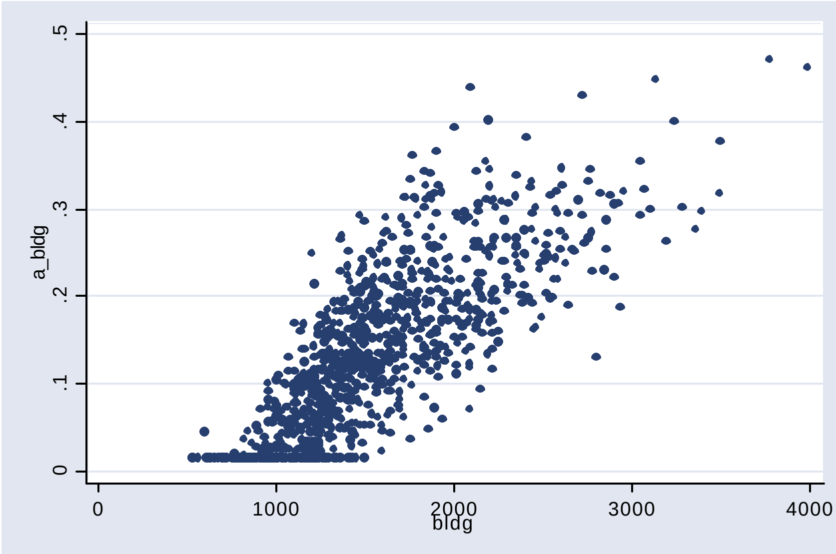

Figure 4b. Sorting on Building Size: Marginal WTP (at mean) matched to Amenity Levels

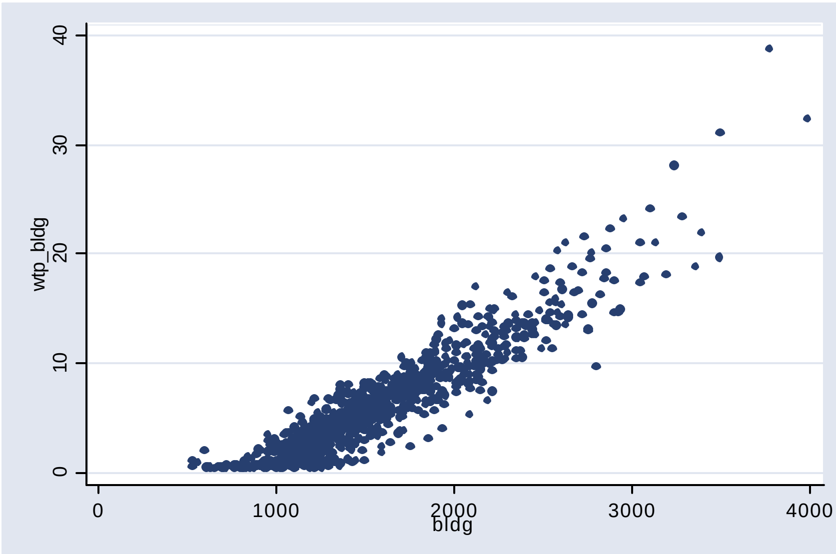

Figure 5. Sorting on Teachers: Marginal WTP (at mean) vs. Amenity Levels

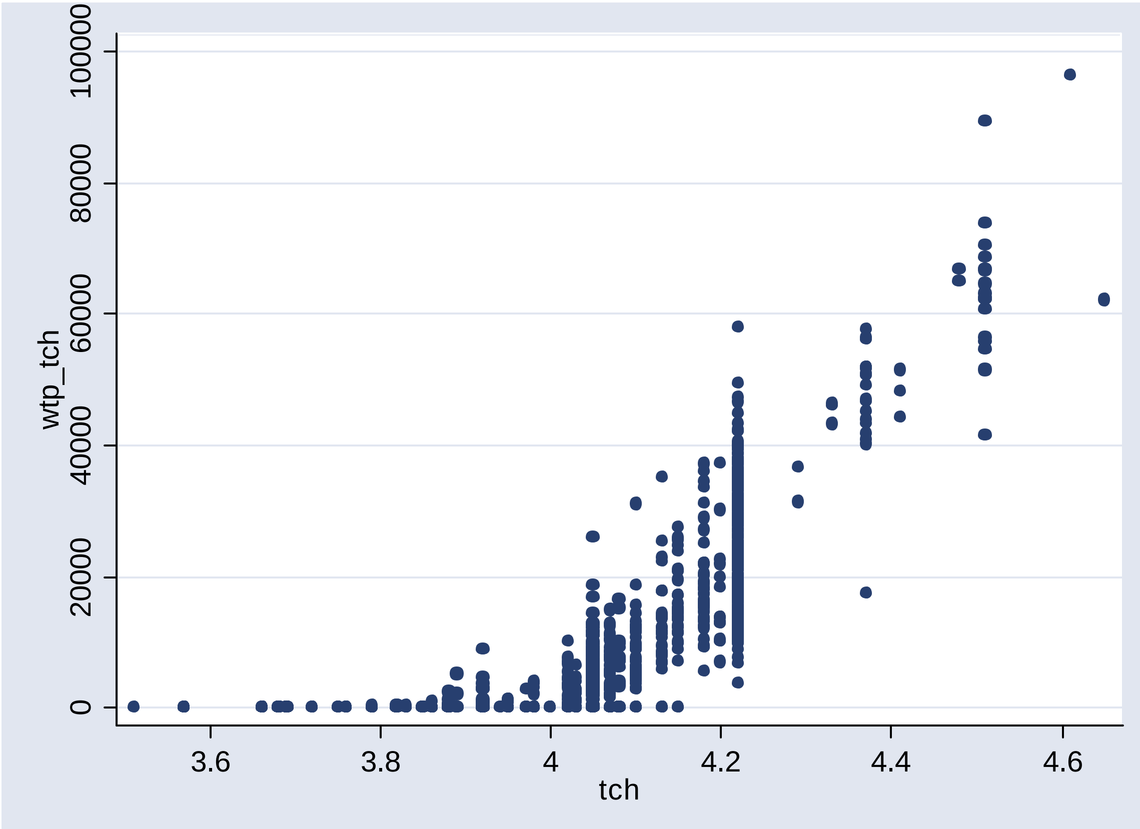

Figure 6. Sorting on Air Quality: Marginal WTP (at mean) vs. Amenity Levels

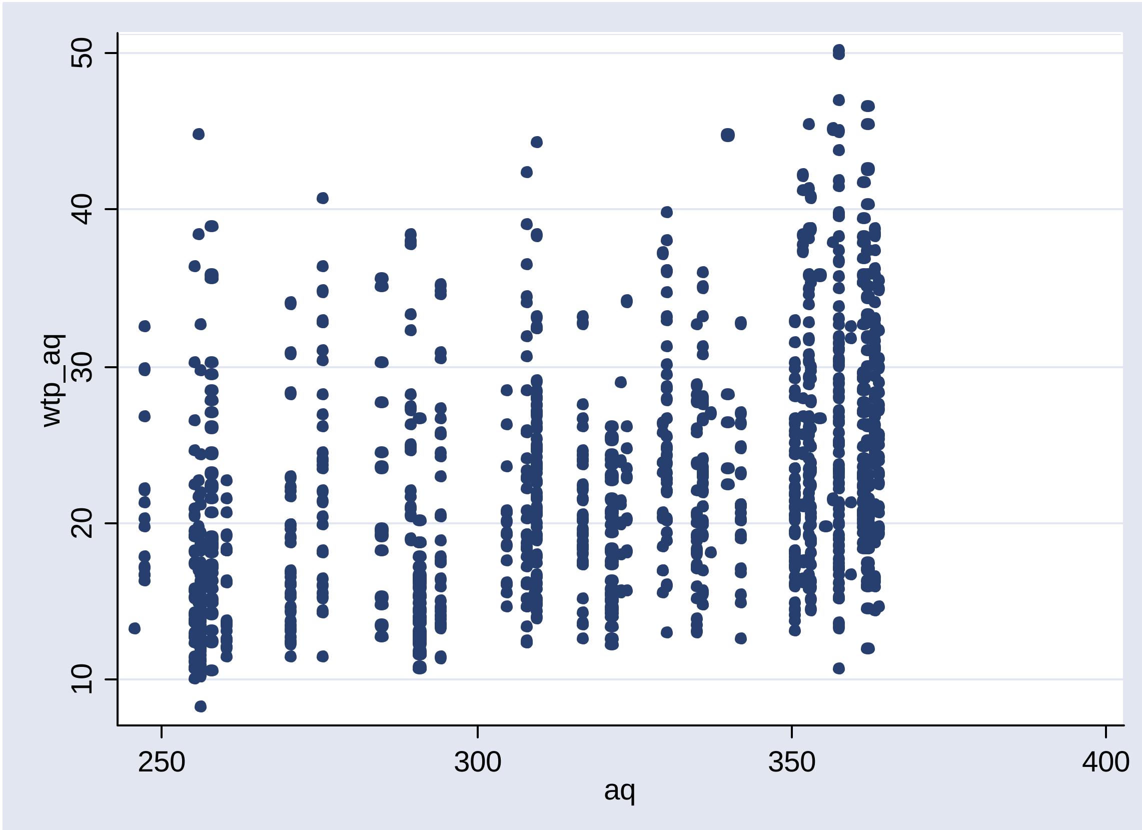

Figure 7. Sorting on Bathrooms: Marginal WTP (at mean) vs. Amenity Levels